

Let  $1, 2, \dots, 100$  be the passengers in boarding order, and let  $1_s, 2_s, \dots, 100_s$  be their assigned seats.

Then we can see that for passenger 1, there are three options. Sitting in  $1_s$  guarantees a success; sitting in  $100_s$  guarantees a failure; sitting in any other seat leaves it up to later passengers. Then there is a  $1/100$  chance of failure, and a  $1/100$  chance of success at this point.

Suppose they do not sit in  $1_s$  or  $100_s$ , instead sitting down in seat  $n_s$ . Then all passengers will find their assigned seats properly, until passenger  $n$ . Passenger  $n$  is now like passenger 1, but on a smaller airplane. They have the instant failure seat,  $100_s$ , and the instant success seat,  $1_s$ , both with equal odds of being chosen. If they instead end up in seat  $m_s$ , then as before, person  $m$  becomes the new "passenger 1".

Then as people make their way in, we have a series of "passenger 1's", who all have equal probability of success or failure, and if no passenger sits in either  $1_s$  or  $100_s$ , then we will get down to passenger 99, who must choose between the two. So at any point, the odds of success or failure are equal, and choosing another seat simply shifts the person who makes the decision. So the probability is  $\frac{1}{2}$ !