

Assuming all sentences are unique and that every number appears at least once, we find that the right hand column is an integer vector with positive entries, each between 1 and 9, whose entries sum up to 20.

Say that 1 appears x times, with $x > 1$. In our right hand column, we know there are $(x-1)$ entries of value 1, 1 entry of value x , and $(10-x)$ entries of unknown value but which must be greater than 1. This means that the column sum is greater than or equal to $(x-1)+x+2(10-x)$, giving us a column sum of at least 19. From this, we know that if 1 appears x times, $(10-x)$ numbers must appear >1 times, and if each appears only 2 times, the column sum will add up to only 19, one less than what we need. This implies the following:

If 1 appears x times, $(x-1)$ numbers must appear 1 time, $(9-x)$ numbers must appear 2 times, and 1 number must appear 3 times, giving a right hand column only containing the values 1,2,3, and x . As a result, the number 2 will appear in the left hand column once and the right hand column $(9-x)$ times, or $(10-x)$ times, which will be equal to 1, 2, or 3, giving us possible values $x=7, 8, 9$. For $x=8$ or $x=9$, counting the numbers appearing does not give a right hand vector with the correct distribution, but when $x=7$, we get the following:

0 appears 1 time.
1 appears 7 times.
2 appears 3 times.
3 appears 2 times.
4 appears 1 time.
5 appears 1 time.
6 appears 1 time.
7 appears 2 times.
8 appears 1 time.
9 appears 1 time.

However, we do not specify that the sentences must be unique, giving us an alternate solution:

0 appears 1 time.
3 appears 1 time.
4 appears 1 time.
5 appears 1 time.
6 appears 1 time.
7 appears 1 time.
8 appears 2 times.
8 appears 2 times.
9 appears 2 times.
9 appears 2 times.

By choosing to duplicate numbers other than 8 and 9, we find that there are at least several trivial variations on this solution. It is unclear to me as to whether "Each number is used at least once"

means that we make a statement about each number once, or that each number must appear at least once.